where C is the coordinate compensation factor. Comparison of Eqs. (5) and (4) shows that, for the case of ignition or burning of coal particles or droplets under finite chemical kinetics,

$$C=I/(d\theta/d\xi)^* \tag{6a}$$

More generally, if η is an arbitrary coordinate in which the Dirac-delta approximation is introduced, then

$$C=I/(d\theta/d\eta)^* \tag{6b}$$

Using polynomial and rational approximations for the first exponential integrals[8] and for the high-activation energy of the reaction,

$$C=\theta^*/(d\theta/d\xi)^* \tag{7}$$

Then if ξ is the coordinate other than θ, the factor C will depend on the gradient $(d\theta/d\xi)^*$. According to previous literature,[2,3] the coordinate compensation factor will be unity for any coordinate in which the Dirac-delta approximation is introduced, whereas the present analysis shows a dependence of the factor on the thermal gradient, which consequently serves to maintain the invariance of the physical quantity. Thus, for the ignition of a droplet or a volatile coal particle, where $\theta_w<<\theta_\infty$ and $\theta^*=\theta_\infty$ under adiabatic ignition conditions,

$$C=2\theta_\infty/[Q_w/(E_g/R)c_p] \tag{8}$$

whereas for a heated carbon particle burning in cold surroundings, where $\theta_w>>\theta_\infty$ and $\theta^*=\theta_w$,

$$C=\theta_w(e^{\xi_w}-1)/(\theta_w-\theta_\infty)\simeq\xi_w \quad \text{for } \xi_w<<1 \tag{9}$$

III. Results

A numerical solution for the burning rate of a heated carbon particle in cold surroundings was carried out.[9] The chemical kinetics of the process was considered to be a finite two-step oxidation process in which C oxidizes to CO on the particle surface and CO oxidizes to CO_2 in the gas phase. The solution revealed a maximum in the burning rate at a particular particle temperature. Then the variation of this particle temperature with respect to the particle size and ambient pressure was studied. In order to establish a correlation of the results in terms of the particle size and ambient pressure with a single dimensionless group, a Dirac-delta approximation was introduced following the argument introduced in the present technical note. The results of the calculations for the variation of the temperature at the peak burning rate with the particle size and ambient pressure are presented in Fig. 1. The solid line presents results from the approximate theory and the symbols the numerical calculations. The agreement between the two is excellent.

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Condensed Phase Heat of Reactions of Phenol-Formaldehyde Composite Propellants

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Introduction

THERE exists sufficient evidence[1,2] to indicate the possibility of some exothermic solid-state reactions in composite propellants. Thus, as in the case of double base propellants, the energy necessary to heat the composite propellant surface is supplied in part by these exothermic reactions and in part by energy transferred from the hotter flame zone. Since most jet propulsion devices operate at comparatively low pressure where only a small amount of energy returns from the flame, these exothermic reactions occurring in the condensed phase are of practical importance.

Recently Kirby and Suh[3] have reported that there exists no reliable method of direct measurement of the condensed phase heat of reactions of solid propellants. However, they suggested an indirect method of computation based on the theory of Adams[4] and experimental technique of Suh et al.[5] The same has been adopted in this communication.

Experimental

For a solid undergoing several consecutive reaction steps to give gaseous products where the first of these is an exothermic zero-order reaction, Adams[4] has shown that heat Q for this initial reaction is

$$Q=\bar{C}_p\ (T_s'-T_o) \tag{1}$$

where $\bar{C}_p$ and T_o are the average specific heat and initial temperature of the propellants respectively. T_s' is the minimum surface temperature at the solid surface with no heat feedback from the gas phase. His analysis further predicted that at T_s' the propellant should burn at a rate approaching infinity.

Recently Girdhar et al.[5] have studied the effect of initial temperature on burning rates of composite propellants by making the measurements in an air thermostat maintained at the desired initial temperature. The experimental data were found to fit the relation[6]

$$r=Cp^n/(T_s-T_o) \tag{2}$$

The surface temperature, T_s was calculated from the slope and intercept of the plot of $1/r$ vs T_o. It can be seen from Eq.

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Table 1 Combustion properties of the propellants

Propellant system	Oxidizer (wt %)	C_p[a] (cal/g°C)	T_s^6 (°C)	Q(condensed phase) (cal/g)	Burning rate[6] at 25°C (cm/sec)
P. F./ammonium perchlorate	77.5	0.30	840	244	0.181
P. F./potassium perchlorate	75	0.24	605	139	0.275
P. F./sodium perchlorate	73.5	0.29	680	190	0.307
P.F./potassium nitrate	75	0.26	510	126	0.453

[a] The specific heat of propellant was assumed to be (¾ C_p oxidizer + ¼ C_p P. F. polymer); C_p at 25°C of ammonium perchlorate is taken from Ref. 7, of potassium nitrate, sodium nitrate, and P. F. polymer from Ref. 8, and of potassium perchlorate from Ref. 9.

(2) that T_s is the temperature (extrapolated) at which the burning rate would become infinite. Thus the T_s measured in our experiments should be identical with T_s' used in Adams[7] theory.

Discussion

Since the specific heats of these propellants at different temperatures are not known, the values at 25°C were used in the present computation. Further assuming an initial temperature of 25°C, heats of reaction in the solid phase of the propellants were calculated using Eq. (1). The condensed phase heats of reaction along with some other burning characteristics of the propellants are given in Table 1. Since the condensed phase reactions are initiated at comparatively higher temperatures, the significant heat of generation does not occur in the experimental temperature range 10-75°C used for the extrapolation of T_s.

It can be seen from the table that for both the nitrate and the perchlorate types propellant burning rate is inversely related to the condensed phase heat of reaction. This shows that it is the pyrolysis kinetics, but not the heat of the exothermic reactions in solid phase, which is significant in controlling the burning rate of composite propellant at low pressures.

Conclusions

The condensed phase heats of reaction of phenol-formaldehyde composite propellants have been computed. These show no relevant connection with the burning rate at atmospheric pressure.

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Discontinuous Solutions of Three-Dimensional Compressible Stagnation Point Boundary Layers

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Introduction

RECENT developments in space flight have focused attention on the problem of predicting the flowfield on general three-dimensional bodies. The steady laminar compressible three-dimensional forward stagnation-point flow of a gas with constant or variable $\rho\mu$ flows (where ρ and μ are the density and viscosity, respectively) both for nodal ($C=b/a \geq 0$, where a and b are velocity gradients at the stagnation point in the x and y directions, respectively) and saddle ($C<0$) points of attachment has been studied by several authors.[1-3] For constant $\rho\mu$ flows ($\rho \propto T^{-1}$, $\mu \propto T$, $Pr=0.7$, where T and Pr are the temperature and Prandtl number, respectively), Libby[1] found two branches of solutions, namely, continuous and discontinuous branches when $C<0$. He observed that the discontinuous branch solutions have certain interesting features that are not found in continuous branch solutions. For example, the transverse profiles for the discontinuous branch have both reverse flow and velocity overshoot for certain critical values of the saddle points of attachment and mass injection, whereas, for the continuous branch, they have only reverse flow but no velocity overshoot. The physical significance of the discontinuous solution when $C \to 0^-$ is that it may represent the flow due to a "wall jet" directed inward along the y axis on a two-dimensional cylinder.[1] For variable gas properties ($\rho \propto T^{-1}$, $\mu \propto T^{\omega}$, $Pr=0.7$, where ω is the index of the power-law variation of viscosity), the discontinuous branch solutions have not been obtained before. However, the continuous branch solutions have been studied by Wortman et al.[2] and Vimala and Nath.[3]

The aim of the present analysis is to obtain the discontinuous branch solutions of the preceding problem with variable gas properties and mass injection for a saddle point of attachment ($-1<C<0$).

Governing Equations

The governing differential equations in dimensionless form for the steady laminar compressible boundary-layer flow with mass injection of a gas with variable properties in the neigh-

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